Study of Charmonia States in Vacuum and High density medium

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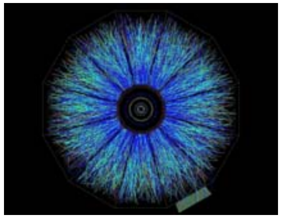
Introduction:

Heavy quarks (charm and bottom) provide a probe for the Quark Gluon Plasma (QGP) because of their heavy masses which are much larger than T_c~.2GeV, the critical temperature for the QGP to form. We study bound states these quarks form, in particular Charmonium, a charm-anticharm bound state. Charmonium and its exited states (J/Ψ, Ψ',…) are studied both in vacuum and in medium (the QGP).

The Quark Gluon Plasma:

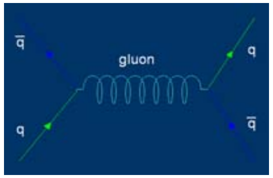
Quantum Chromodynamics (QCD) predicts an exotic state of quark matter at temperatures of about 2*10^{12} K (170 MeV), the Quark Gluon Plasma (QGP) . Experiments performed at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, where two Au nuclei are sent to collide at relativistic energies provide evidence for such a medium to occur.



The kinetic energy of the colliding Au Nuclei is converted into compression and thermal energy which allow different matter types to be produced. After the collision, pions and other hadrons containing u, d, s, quarks, are observed. It is believed tough that the life time of the QGP is just 10^{22} s after the collision. Nevertheless this interval of time is enough for this "fireball" to reach equilibrium. Hadrons containing heavy quarks (charm and bottom) have been identified as possible probes for the QGP. This follows from the charm-quark and bottom-quark masses m_c~1.5GeV/c^2 and m_b~4.5GeV/c^2 which are larger than the typical temperatures T_c of the QGP which are not expected to thermalize from the process[1].

Quarks:

Quarks are elementary particles, that interact via the four fundamental forces. Due to their color charge (Strong Force), they are confined to one another. During Collisions, quarks are released from the hadrons they are confined to, and form new color neutral states. Force carries for color interaction are gluons which also interact with themselves because they also carry a color charge.



Procedures:

We modeled the Charmonium system using the Schrodinger model. The large quark mass enables us consider a non-relativistic approach to the problem in which we can describe the interaction by a potential. The potentials were a "Cornell Potential",(described on the right), and a "Screening Cornell" (farther right). For this, Schrodinger's equation was solved. Here m is the reduced mass of the particle interacting with a potential V.

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t)$$

Time independence are considered, thus Ψ becomes a function of only r, θ and φ [2]. Since we have a central potential we can solve and get a solution in the form Ψ(r, θ, φ)=R(r)*Θ(θ)*Φ(φ), were the equation for R(r) is:

$$\left(\frac{1}{r^2}\right)\frac{d\left(r^2\frac{dR(r)}{dr}\right)}{dr} + \left(\left(2*\frac{m}{\hbar}\right)*(V(r)+E) - l*\frac{l+1}{r^2}\right)*R(r) = 0$$

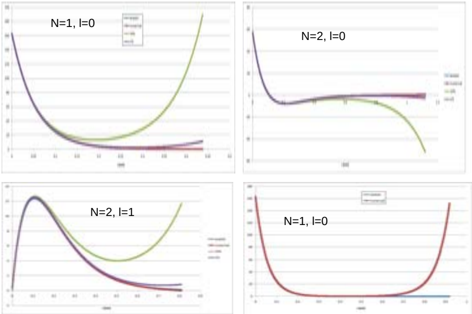
The equation was solved by numerical methods with different potentials. In each case different "Eigen Energies" and wave functions were obtained. Everything was programmed in Fortran 77

Hydrogen:

In order to test numerical precision and because of its simplicity and similarity to the Cornell potential, the coulomb's potential was implemented at first.

$$V(r) = -\frac{\propto}{r}$$

The following plots display the radial wave functions for the Hydrogen Atom. The analytic solution , the numerical solution, and solutions with the Eigen energies changed by 1% and 10% are plotted.



Graph set 1. The first three graphs show the normalized wave functions for the first energy levels hydrogen. The fourth one

As can be seen from the graphs, the analytic and numerical solution cannot even be differentiated by the eye from our resolution.

The fourth graph, shows the divergence of our numerical solution for N=1, l=0 after about 0.5 nm where our solution cannot be trusted anymore. This occurs because it stops following the real behavior of a wave function due to accuracy loss at large r-values, were it is supposed to go to zero as the radius increases without bound.

We introduce notation:

- "l" is the angular momentum quantum number
- We call Eigen Energy the Eigen Value of a solution of the radial equation and we interpret this Energy as the sum of the Binding and Kinetic Energies of the system.
- If the system is described by a quantum number "l", then there is an element of a discrete set of numbers that together with "l" describes a unique configuration of the system. Furthermore we assign the value n=l+1 to the lowest energy state corresponding to "l", and n=l+k to the kth state after this one in ascending order.

Cornell Potential 1:

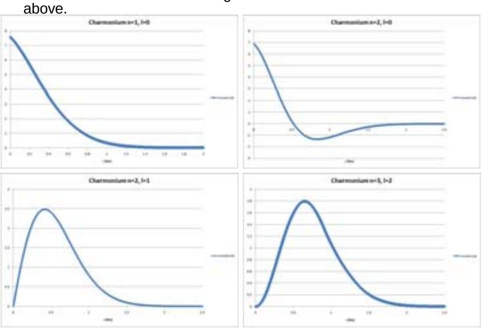
A Cornell potential is implemented

$$V(r) = -\frac{4\propto}{3\,r} + \sigma r$$

α=.212
σ=.42^2 GeV^2

The potential 's 1/r term is called a "Coulomb" term and accounts for the one-gluon exchange interaction, which is not related to any E&M interaction. The second (linear) term, is the confinement term which accounts for quark confinement.

The following graphs show the wave functions for the main states of charmonium according to the notation described above.

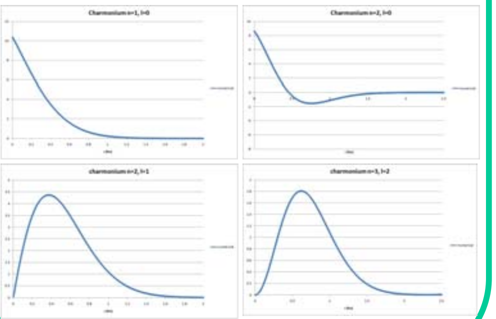


Graph set 2. The graphs show the first states for charmonium using Cornell potential 1.

Cornell Potential[3] 2

$$:V(r) = -\frac{\propto}{r} + \sigma r$$

α=.52
σ=.1826 GeV^2



Graph set 3. The graphs show the first states for charmonium using Cornell potential 3.

For each potential a different charm rest mass was used, this to match the ground energies to about 3.096 GeV, the rest mass of charmonium. The energies obtained are the following: (Format: charm mass, System mass, Eigen Energy) (System mass=Eigen Energy +2*mass). Energy in GeV's

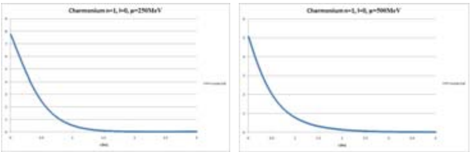
Rest mass = 1.2851 3.0969 E 1 = 0.5267 l= 0	Rest mass = 1.36492 3.0969 E 1 = 0.3671 l= 0
3.6594 E 2 = 1.0893 l= 0	3.7152 E 2 = 0.9854 l= 0
3.4514 E 2 = 0.8812 l= 1	3.5289 E 2 = 0.7991 l= 1
3.69855738 E 3 = 1.1283 l= 2	3.83186436 E 3 = 1.1020 l= 2

Color Screening potential:

Interacting matter of sufficient temperature and pressure is predicted to undergo a transition to a state of deconfined quarks and gluons (the QGP). We say deconfinement occurs when color charge screening becomes strong enough that it shields the quark binding potential with any other quark or anti quark[4]. Parameters were changed so that they matched those from [4]. We model this effect using the following potential with the following parameters :

$$V(r,T) = \left(\frac{\sigma}{\mu(T)}\right)\left(1 - e^{-\mu(T)*r}\right) - \left(\frac{\alpha}{r}\right)e^{-\mu(T)*r}$$

r radius
μ(T) temperature dependent "Screening mass"
σ 0.192 GeV^2
α 0.471



Preliminary data shows how the wave function spreads as the screening mass is increased, which matches the prediction on which the quarks will dissociate after certain temperature is reached. Also, as we can see from the graphs, the wave functions are more spread than those were we use our vacuum potentials. These are the Energies of the presented states above:

Rest mass = 1.320 Mu=250MeV
J/psi mass=3.0440
E 1 = 0.4040 l= 0

Rest mass = 1.320 Mu=500MeV
J/psi mass=2.9836
E 1 = 0.3435 l= 0

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References:

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